CHECKING THE VALIDITY OF THE TOLL COLLECTORS' VEHICLE CLASSIFICATIONS

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INTRODUCTION

THE PROBLEM

At toll facilities that offer reduced rates to commuters a question which often arises is, "How do you know that some collectors are not registering non-commuter vehicles at the reduced commuter rate and then pocketing the difference"? The answer usually given is that auditing and visual controls are employed to guard against this eventuality.

A daily audit of each toll collector's deposit report is made to determine whether sufficient cash or its equivalent has been deposited to cover the transactions appearing on his register tape. This daily audit is supplemented by an occasional detailed analysis of the commutation tickets turned in by each collector. The object of the latter audit is to determine whether a collector turned in too many tickets from one book.

In both of these audits the analysis is confined to what a specific collector did without considering how the work of one collector compares with another collector when both worked under similar conditions. Questions such as, does one collector generally have more commuter vehicles than the other collectors, are not evaluated. At toll facilities that do not use commutation tickets primary control is attained by visually inspecting the toll transactions.

As the vehicles pass through the toll lanes the supervisor on duty verifies that the vehicles are being properly recorded. Theoretically, visual inspection provides for a first-hand check on the toll collectors but in actual practice certain impediments arise which nullify any systematized inspection procedure. The foremost of these is the monotony of the inspection process. Other factors are; weather conditions, heavy traffic flow, accidents and customers problems.

The purpose of this paper is to demonstrate a general method of analysis that will augment the auditing process by taking into consideration not only the toll transactions registered by the collector but also will consider the likelihood of a collector having misclassified a portion of the vehicles. The method also eliminates the necessity of depending on constant visual inspection as a contol technique.

Introduction to Statistical Control

After the auditing department has determined that all the collectors are in balance (i.e. their deposit reports balance with their register tapes) a statistical analysis is begun. The commuter and non-commuter passenger cars are entered on work sheets similar to table 1. A separate work sheet is used for each day of the week and each shift because of varying traffic characteristics.[1]

The commuter and non-commuter vehicles are not analyzed separately but are combined into a single statistic, the per cent of commuters of the total passenger cars i.e.

$$\Sigma \frac{\Sigma \text{ commuters}}{\text{commuters} + \Sigma \text{ non-commuters}} 100.$$

This transformation is made because of daily fluctuations in each class and more importantly because of the existence of a relationship between the classes. If a toll collector deliberately registers a non-commuter as a commuter he reduces the number of non-commuters by one but he also increases the number of commuters by one. The net effect is an increase in his per cent of commuters.

[1]The discussion that follows will be restricted to passenger cars. The procedures are applicable to all types of vehicles however.

TABLE 1: COMMUTER AND NON-COMMUTER PASSENGER CARS
MONDAYS ONLY
3 P. M. - 11 P. M. - SOUTHBOUND TRAFFIC
MYSTIC RIVER BRIDGE
MARCH THRU MAY

	Lane 10			Lane 11			Lane 12			Lane 13		
Wk.	Commuter	Non-Comm.	% Comm.	Commuter	Non-Comm.	% Comm.	Commuter	Non-Comm.	% Comm.	Commuter	Non-Comm.	% Comm.
1	1061	382	73.5	1419	371	79.3	1610	401	80.1	1489	237	86.3
2	960	250	79.3	1350	316	81.0	1547	321	82.8	1203	170	87.6
3	997	363	73.3	1400	361	79.5	1648	353	82.4	1493	222	87.1
4	1056	376	73.7	1489	469	76.0	1635	342	82.8	1755	299	85.4
5	1108	407	73.1	1410	452	75.7	1691	374	81.9	1511	245	86.0
6	1178	406	74.4	1352	444	75.3	1739	360	82.8	1446	216	87.0
7	1105	409	73.0	1571	498	75.9	1669	362	82.2	1452	268	84.4
8	1179	951	55.4	1543	870	63.9	1585	612	72.1	1502	517	74.4
9	1063	565	65.3	1678	647	72.2	1561	449	77.7	1771	313	85.0
10	1292	579	69.1	1789	659	72.7	1710	491	77.7	1834	378	82.9
11	1304	607	68.2	1627	744	68.6	1763	575	75.4	1737	416	80.7
12	1164	585	66.6	1681	707	70.4	1620	575	77.0	1880	424	81.6
13	1248	616	67.0	1855	733	71.7	2020	623	76.4	1743	326	82.3

Reference to table one and the column headed "per cent of commuters" reveals two important characteristics:

1. The per cent of commuters tend to decrease with the passage of time. This is due mainly to seasonal variation.
2. When the per cent of commuters decrease or increase from week to week the same pattern generally occurs in all the lanes.[2]

With the approach of spring and improved driving conditions not only does the volume of traffic increase but the length of the average trip increases. The non-commuter vehicles consequently increase at a faster rate than the commuters and this in turn causes a decrease in the per cent of commuters. After Labor Day the reverse of this is true.

[2]In some cases where the change is very small this observation may not hold.

Whereas toll facilities are open 24 hours a day, seven days a week, the amount of data to be deseasonalized is extremely large. Removing the effects of seasonal variation would be very time consuming and costly. Therefore, a method which circumvents the necessity of removing seasonal is utilized.

Since the data is collected on a same day, same tour, same direction basis a good comparison of each toll collector's work is obtained if the per cent of commuters value of each lane is matched against the value that occurred in that lane last week. (To avoid the possibility of always matching the same collectors alternate assignments are used.)[3] The difference and sign of each matched pair is now determined and then these differences are ordered. For example, when week 2 is matched against

[3]See appendix for toll collector lane assignments.

week 1, the differences and assigned ranks are:

Lane	10	11	12	13
Difference	+05.8	+01.7	+02.7	+01.3
Rank	1	3	2	4

The largest positive difference is assigned a rank of one (1) and the smallest positive difference is assigned a rank of four (4). When ties occur, the two differences are given the average rank (i.e. if the two differences are tied for ranks 3 and 4 the average of these ranks is given to each difference)[4] Table 2 shows the differences and assigned ranks for a thirteen (13) week period.

The reader may question the use of ranks on the basis that a certain amount of information is lost. While this is true, the advantage of ranking the differences is that on an information per dollar of cost basis we come out ahead. Other advantages are:

1. The order of the differences is more important than the quantitative value of the differences. The essential point here is that in one case the size of a particular difference may be unimportant while in another case a difference of the same size may be extremely important.

2. Ranking allows persons who are unfamiliar with more technical statistical procedures the opportunity to evaluate this data.

3. Ranking lends itself to machine data processing.

<u>Distribution of the ranks in the toll lanes</u>

As noted earlier, the toll collector assignments are made on a prearranged basis so that the same collectors will not always be matched against each other. In an attempt to make the conditions under which the matching occurs as identical as possible and also to allow only chance variables to influence the flow of traffic, all collectors must use a standard method of collecting tolls. Each vehicle is classified before the fare is collected and each collector carries money in his hand for the purpose of making change. At this point it is of considerable interest to inquire whether the toll lanes themselves influence the rank assignments.

A review of the rank assignments in table 2 indicates that all lanes received the different ranks about an equal number of times. But since this table represents only thirteen (13) weeks, conclusions based on such a small sample could be erroneous. However, the same conclusion resulted when a random sample of two hundred matched days was drawn from a population of two years. The sum of the ranks for this sample was:

Toll lane	10	11	12	13
Sum of the ranks	483.0	523.5	490.5	503.0

Substituting these rank totals in the Friedman two-way analysis of variance by ranks formula

$$X_r^2 \quad \frac{12}{Nk\ (k+1)} \sum_{j\ \ 1}^{k} (Rj)^2 - 3N\ (k+1)$$

where N the number of rows
k the number of columns
Rj the sum of the ranks in each column we obtain:

$$X_r^2 \quad \frac{12}{200\ (4)\ (5)} \left[(483.0)^2 + (523.5)^2 + (490.5)^2 + (503.0)^2 \right] - 3\ (200)\ (5) \quad 2.68$$

Since the Friedmen statistic tends to be distributed as Chi-Square with k-1 degrees of freedom, reference to a table of Chi-Square indicates that $p > .30$ but $< .50$.[5] Since $p > .05$ (the level of significance) we can conclude that the rank assignments are independent of the toll lanes. Stated another way the mean of the rank total for each lane will be about equal. When the mean rank of each toll lane is computed in the 200 day matched sample we obtain;

Lane	10	11	12	13
Mean Rank	2.41	2.62	2.45	2.52

It will be noted that the sample means differ only slightly from the hypothetical mean of 2.5 (the value

[4] Friedman, M., "The use of ranks to avoid the assumption of normality implicit in the analysis of variance,"Journal of American Statistical Assn,32(1937)675-701

[5] Friedman, op. cit. 675-701

TABLE 2: DIFFERENCES AND RANKS OF THE PER CENT OF COMMUTERS

DATA FROM TABLE 1

	Lane 10			Lane 11			Lane 12			Lane 13		
Wk.	% Comm.	Differ- ence	Rank	% Comm.	Differ- ence	Rank	% Comm.	Differ- ence	Rank	% Comm.	Differ- ence	Rank
1	73.5			79.3			80.1			86.3		
2	79.3	05.8	1	81.0	01.7	3	82.8	02.7	2	87.6	01.3	4
3	73.3	-06.0	4	79.5	-01.5	3	82.4	-00.4	1	87.1	-00.5	2
4	73.7	00.4	1.5	76.0	-03.5	4	82.8	00.4	1.5	85.4	-01.7	3
5	73.1	-00.6	3	75.7	-00.3	2	81.9	-00.9	4	86.0	00.6	1
6	74.4	01.3	1	75.3	-00.4	4	82.8	00.9	3	87.0	01.0	2
7	73.0	-01.4	3	75.9	00.6	1	82.2	-00.6	2	84.4	-02.6	4
8	55.4	-17.6	4	63.9	-12.0	3	72.1	-10.1	2	74.4	-10.0	1
9	65.3	09.9	2	72.2	8.3	3	77.7	05.6	4	85.0	10.6	1
10	69.1	03.8	1	72.7	00.5	2	77.7	00.0	3	82.9	-02.1	4
11	68.2	-00.9	1	68.6	-04.1	4	75.4	-02.3	3	80.7	-02.2	2
12	66.6	-01.6	4	70.4	01.8	1	77.0	01.6	2	81.6	00.9	3
13	67.0	00.4	3	71.7	01.3	1	76.4	-00.6	4	82.3	00.7	2
			29			31			31			29

which would be obtained if the rank totals of all the lanes were equal). The hypothetical mean or average rank is also the value that is used as a standard to evaluate the ranks received by the individual collector. Over a period of time each collector can be expected to receive all the rank values about an equal number of times assuming only chance variations.

Evaluating the ranks of the individual collector

From a control point of view primary interest centers on those collectors who receive an abnormal number of below average ranks.[6] To illustrate this point consider the case of collector Y who worked in lane 12 during the fifth week. From table one it is noted that he registered 1,691 commuter vehicles and 374 non-commuter vehicles. Based on the differences that occurred in the other lanes, collector Y received a rank of four (4). However, suppose collector Y had deliberately misclassified 20 non-commuters as commuters. His register tape would then indicate 1,711 commuters and 354 non-commuters. The differences and ranks would then be;

Lane	10	11	12	13
Difference	-00.6	-00.3	-00.1	+00.6
Rank	4	3	2	1

The conversion of the 20 non-commuters into 20 commuters would have changed Y's rank from four (4) to two (2) and would place him below the standard on this day. The question of how many below average ranks does a collector have to receive before any action is taken will now be considered.

[6] If a collector receives the average rank several times in a sample, half of them are counted as below and half above the average rank.

Choosing a level of significance

Rather than arbitrarily selecting the level of significance at .01 or .05 consideration must be given to the consequence of committing either a type I or type II error.

In the case under discussion a type I error occurs when we reject the null hypothesis that a collector is honest when in fact he is honest. What is the cost of committing this type error? It would be the cost of conducting a visual investigation of the toll collector in question and this would be a relatively small cost. A type II error occurs when we accept the null hypothesis that the collector is honest when in fact he is dishonest. The cost of committing the type II error is the retention of the dishonest toll collector and this could be expensive over a period of time.

When the dollar costs of committing these errors are weighed, it is quite evident that we particularly want to avoid a type II error. By selecting .15 level of significance we attempt to minimize the cost of committing both types of error.

On this basis we may determine whether any collector is statistically out of control. This is accomplished by noting how many below average ranks a toll collector received in a given sample (usually the size of the sample is > 25) and then noting whether the value equals or exceeds the critical value given in the following table.[7]

Table 3. Critical number of below average ranks in a given sample.

Sample size	Critical value
26	17
27	17
28	18
29	18
30	19
31	19
32	20
33	20
34	21
35	22
36	22
37	23
38	23
39	24
40	24

[7]These values were determined by using the normal approximation to the binomial distribution corrected for continuity, with $P = Q = 1/2$

A case history

In order to indicate the actual procedure which is followed in checking the validity of the toll collector's vehicle classifications, a case history is submitted.

Prior to starting his regular toll collecting duties, collector A was given the standard indoctrination. This consists of instructions on the traffic rules & regulations of the facility, plus a period of actual toll collecting under the supervision of a toll sergeant. At the completion of this training, collector A was assigned a regular tour of duty.

During his first three weeks of toll collecting he received the following ranks:

Day	Rank	Day	Rank
1	5	9	4
2	2	10	3
3	3	11	1
4	4.5	12	1
5	4	13	2
6	2.5	14	5
7	5	15	1
8	5		

A review of these ranks indicate that on eight days he was above the average rank and on seven days he was below. This is of course a perfectly normal situation.

The ranks received by collector A in the next eleven days were:

Day	Rank	Day	Rank
16	1	22	1
17	1	23	1
18	1	24	1
19	2	25	1
20	4	26	1
21	1		

When these ranks are added to the ones he previously received the results are;

above the average rank 9 days
below the average rank 17 days

Referring to table 3 we note that in a sample of 26 days, 17 below average ranks is critical. A non-statistical investigation of collector A's work was initiated at this point.

When a visual inspection of collector A's work was made it was noted that he was misclassifying non-commuter vehicles at the reduced commuter rate. On the basis of this evidence collector A became an "X" collector.

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APPENDIX

Toll collectors lane assignments, Schedules A and B.

To avoid the possibility of always comparing the same two collectors, an alternating schedule is used. Schedule A is in effect during the odd numbered weeks and B during the even numbered weeks.

SCHEDULE A

Time	Monday South	Monday North	Tuesday South	Tuesday North	Wednesday South	Wednesday North	Thursday South	Thursday North	Friday South	Friday North	Time	Saturday South	Saturday North	Time	Sunday South	Sunday North
11-7	BB-10 (1)-11	DD-5 CC-4	AA-10 EE-11	BB-4 CC-5	AA-11 BB-10	EE-5 *-4	AA-10 DD-11	BB-5 EE-4	EE-10 CC-11	DD-5 AA-4	11-7	EE-11 DD-10	CC-4 AA-5	11-7	(6)-11 CC-10	DD-5 BB-4
6-2	B-12 K-13	C-6	C-12 A-13	B-2	A-12 B-13	C-6	B-12 C-13	A-2	C-12 A-13	B-3						
											6:30-2:30	A-12				
7-3	J-10 H-11 I-9	G-3 F-4 E-5	H-9 G-10 E-11	M-3 F-5. I-4	M-11 E-10 F-9	G-3 H-4 L-5	F-10 K-11 L-9	E-3 M-5 J-4	I-10 J-9 M-11	*-4 L-6 K-5	7-3	(2)-11 G-13 K-9 J-10	H-6 I-5 L-4	7-3	I-13 F-11 K-10	E-5 G-4 J-6
8-4		D-2		D-6		D-2		D-6		D-2	8-4		M-2	8-4	L-12	H-3
											9-5	(3)-3		2-10	S-9	R-2
2-10	Q-13 R-12	O-6	O-12 R-13	Q-2	O-13 Q-12	R-6	R-12 Q-13	O-2	Q-13 O-12							
3-11	V-11 X-10	S-5 U-4 W-3	U-10 N-11	S-3 V-5 T-4	S-10 U-11	T-3 N-5 Y-4	N-10 W-11	X-4 Y-3 S-5	*-11 Y-10	V-6 W-4 N-5	3-11	V-10 W-9 T-11 Y-13	N-6 U-5 X-4	3-11	U-13 W-11 X-10	V-6 Y-4 T-5
4-12	P-9	T-2	Z-9	P-6	P-9	Z-2	Z-9	P-6	X-9	Z-2	4-12		Z-2	4-12	(8)-12	(7)-3
5-1										P-3	5-1	(5)-12	(4)-3			
Time	Monday		Tuesday		Wednesday		Thursday		Friday		Time	Saturday		Time	Sunday	

*SERGEANT WORKS THIS TOUR

SCHEDULE B

Time	Monday South	Monday North	Tuesday South	Tuesday North	Wednesday South	Wednesday North	Thursday South	Thursday North	Friday South	Friday North	Time	Saturday South	Saturday North	Time	Sunday South	Sunday North
11-7	DD-11 (1)-10	BB-5 CC-4	BB-11 AA-10	EE-4 CC-5	*-11 AA-10	BB-4 EE-5	EE-11 DD-10	BB-5 AA-4	DD-11 EE-10	AA-5 CC-4	11-7	CC-11 EE-10	DD-5 AA-4		CC-10 BB-11	(6)-5 DD-4
6-2	K-12 C-13	B-2	A-12 B-13	C-6	B-12 C-13	A-2	C-12 A-13	B-6	A-12 B-13	C-3						
											6:30-2:30	A-13				
7-3	E-10 F-11 G-9	H-3 I-4 J-5	I-9 F-10 M-11	E-3 G-5 H-4	E-9 G-11 H-10	F-3 L-4 M-5	F-10 M-9 J-11	E-5 K-4 L-3	M-9 L-11 K-10	I-5 *-4 J-2	7-3	(2)-12 I-10 H-11 L-9	G-4 K-5 J-6	7-3	E-10 G-13 J-11	I-6 F-4 K-5
8-4		D-6		D-2		D-6		D-2		D-6	8-4		M-2	8-4	H-12	L-3
											9-5		(3)-3	2-10	R-9	S-2
2-10	Q-12 O-13	R-2	Q-13 R-12	O-6	O-12 R-13	Q-2	Q-12 R-13	O-6	Q-13 O-12							
3-11	X-11 S-10	V-4 U-3 W-5	S-11 V-10	U-5 T-3 N-4	N-10 U-11	T-4 S-5 Y-3	Y-11 S-10	W-4 X-5 N-3	W-10 *-11	V-5 Y-2 N-4	3-11	N-10 T-9 U-13 X-11	V-4 W-6 Y-5	3-11	U-11 V-13 Y-10	T-4 W-5 X-6
4-12	T-9	P-6	P-9	Z-2	Z-9	P-6	P-9	Z-2	Z-9	X-6	4-12		Z-2	4-12	(7)-12	(8)-3
5-1										P-3	5-1	(4)-12	(5)-3			
Time	Monday		Tuesday		Wednesday		Thursday		Friday		Time	Saturday		Time	Sunday	

*SERGEANT WORKS THIS TOUR